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ESTIMATING K_α FOR USE IN EVALUATING CYCLIC RESISTANCE OF SLOPING GROUND[a]

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ABSTRACT

The recently formulated relative state parameter index ξ_R was shown to be a practical index for describing the experimentally observed variation of K_α with both relative density and effective confining stress (Boulanger 2003a). This paper describes the subsequent development of relations that can be used to estimate K_α as a function of both relative density and effective confining pressure. These relationships are also expressed in terms of modified SPT blow count $(N_1)_{60}$ and normalized CPT tip resistance q_{c1N} to facilitate their use in practice for estimating the cyclic resistance of soils beneath sloping ground.

INTRODUCTION

In the initial development of analytical procedures for evaluating the performance of earthdams during earthquakes, the cyclic resistance of the soils comprising the embankment and the foundation was evaluated by conducting cyclic tests on samples of these soils. To accommodate the effects of the initial static shear stress on the cyclic resistance, the cyclic tests were conducted by consolidating the sample under anisotropic loading conditions in a triaxial test prior to applying the cyclic load. Similarly, simple shear tests were conducted by consolidating samples with an initial horizontal shear stress before applying the cyclic load. In both tests, the eventual failure plane had an initial (i.e., static) shear stress, τ_s, and an initial effective normal stress, σ'_{vo}. The ratio $|\tau_s|/\sigma'_{vo}$ has been designated α, and the cyclic resistance for a given number of stress cycles has been related to the initial effective normal stress for various values of α. The earliest tests, which had been conducted on samples of the Sheffield Dam foundation soils (Seed et al 1969), and the subsequent tests on samples of the San Fernando Dams (Seed et al 1975) showed an increase in the cyclic resistance as α increased under all effective confining stresses.

Seed (1983) subsequently introduced the static shear stress ratio correction factor (K_α) as a means for extending the SPT-based liquefaction correlations from level ground conditions to sloping ground conditions. The K_α factor was applied in conjunction with the overburden stress correction factor (K_σ) to adjust the cyclic stress ratio, $(CSR)_{\sigma'_{vo}=1;\alpha=0}$, required to trigger

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liquefaction at α = 0 and σ_{vo}' = 1 tsf (≈1 atmosphere, P_a). The cyclic stress ratio for α > 0 and σ_{vo}' ≠ 1 tsf is then given by:

$$(CSR)_{\sigma_{vo}' \neq 1; \alpha > 0} = K_\alpha K_\sigma (CSR)_{\sigma_{vo}' = 1; \alpha = 0} \tag{1}$$

Harder and Boulanger (1997) summarized the available cyclic laboratory test data and confirmed that K_α depends on both relative density, D_R, and effective confining stress. However, Harder and Boulanger suggested that for an initial effective normal stress less than about 3 tsf (≈ 3 P_a), the variation of cyclic resistance with α depends primarily on D_R, as shown in Fig. 1. For this range of σ_{vo}', the ranges shown in Fig. 1 indicate that the cyclic resistance of dense sands can increase significantly as α increases, while the cyclic resistance of loose sands can decrease significantly as α increases.

Vaid and Chern (1985) conducted two series of cyclic triaxial tests on dense (D_R = 0.7) samples of tailings sand; one series of tests was conducted under an initial confining pressure, σ_{3c}' = 2P_a, and the other series was conducted under an initial confining pressure, σ_{3c}' = 16P_a. The test results showed that the much larger confining stresses produced K_α values that decreased significantly as α increased. The test results conducted with σ_{3c}' = 2P_a produced the opposite trends, i.e., K_α values that increased significantly as α increased. These cyclic triaxial test results will be covered in more detail later in this paper.

This paper describes the derivation of relations that can be used to estimate K_α as a function of both confining stress and denseness (expressed in terms of D_R, normalized SPT blow count, or normalized CPT tip resistance). These relations build on the work of Boulanger (2002, 2003a) who introduced a relative state parameter index, ξ_R, and showed that it could be used as a reasonable and practical index for describing the experimentally observed variation of K_α with both D_R and σ_{vo}'. The specific steps covered in this paper are as follows:

- Review the definition of ξ_R.
- Derive relations between K_α and ξ_R based on cyclic simple shear data at confining stresses of about 2P_a.
- Show that the derived relations satisfactorily describe the effect of high confining stresses demonstrated in the tests of Vaid and Chern (1985).
- Use the relations to illustrate the effects of various parameters on K_α over a broader range of conditions.
- Express the relations in terms of SPT $(N_1)_{60}$ and CPT q_{c1N} to facilitate their implementation in practice.

RELATIVE STATE PARAMETER INDEX

The relative state parameter index, ξ_R, as defined by Boulanger (2002, 2003a) is given by:

$$\xi_R = \frac{1}{Q - Ln\left(\frac{100p'}{P_a}\right)} - D_R \tag{2}$$

in which p' is mean effective normal stress, P_a is atmospheric pressure, D_R is relative density, and Q is an empirical constant. This simple index for representing state was derived from Bolton's (1986) relative dilatancy index. Bolton indicated that the parameter Q depends on the grain type and is approximately equal to 10 for quartz and feldspar, 8 for limestone, 7 for anthracite, and 5.5 for chalk.

Results of cyclic simple shear and cyclic triaxial tests were compiled by Boulanger (2002) and compared in terms of the relative state parameter index, ξ_R, for each test. The relevant test parameters and the value of ξ_R for each test are summarized in Table 1 for the simple shear tests and in Table 2 for the triaxial tests. The test data presented in Tables 1 and 2 indicate that the test conditions that would normally be referenced by an effective normal stress and a relative density need only be referenced by the relative state parameter index, ξ_R. Therefore, the results can now be exhibited in terms of K_α as a function of α for a specific value of ξ_R as shown in Fig. 2 for the simple shear data.

DERIVATION OF RELATIONSHIPS RELATING K_α TO ξ_R

The cyclic simple shear test results shown in Fig. 2 can be interpolated and replotted in terms of K_α versus ξ_R for a selected value of the parameter α. Such plots were prepared at α = 0.05, 0.1, 0.15… 0.35 and used to derive the following general expression relating K_α to ξ_R:

$$K_\alpha = a + b\,exp\left(\frac{-\xi_R}{c}\right) \tag{3a}$$

The parameters (a), (b), and (c) are functions of α and are obtained from the following expressions:

$$a = 1267 + 636\alpha^2 - 634\,exp(\alpha) - 632\,exp(-\alpha) \tag{3b}$$

$$b = exp\left(-1.11 + 12.3\alpha^2 + 1.31Ln(\alpha + 0.0001)\right) \tag{3c}$$

$$c = 0.138 + 0.126\alpha + 2.52\alpha^3 \tag{3d}$$

Equations (2) and (3) can then be used to estimate the static shear stress ratio correction factor K_α for any desired combinations of initial static shear stress conditions and state, expressed in terms of (ξ_R).

TABLE 1
CYCLIC SIMPLE SHEAR TESTS
(After Boulanger, 2002)

Relative Density	σ'_{vc} (atm)	K_o	p' (atm)	τ_{cyc}/σ'_{vc}	α	K_α	ξ_R
Tests on Ottawa Sand Conducted by Vaid & Finn (1979)							
0.68	2	0.45	1.27	0.170	0	1	-0.486
0.68	2	0.45	1.27	0.220	0.093	1.294	-0.486
0.68	2	0.45	1.27	0.247	0.192	1.453	-0.486
0.50	2	0.45	1.27	0.107	0	1	-0.306
0.50	2	0.45	1.27	0.099	0.093	0.925	-0.306
0.50	2	0.45	1.27	0.094	0.192	0.879	-0.306
0.50	2	0.45	1.27	0.096	0.291	0.897	-0.306
Tests on Sacramento River Sand Conducted by Boulanger at al (1991)							
0.35	2	0.45	1.27	0.126	0	1	-0.156
0.35	2	0.45	1.27	0.120	0.100	0.952	-0.156
0.35	2	0.45	1.27	0.091	0.200	0.725	-0.156
0.35	2	0.45	1.27	0.080	0.300	0.635	-0.156
0.55	2	0.45	1.27	0.160	0	1	-0.356
0.55	2	0.45	1.27	0.157	0.100	0.981	-0.356
0.55	2	0.45	1.27	0.166	0.200	1.038	-0.356
0.55	2	0.45	1.27	0.183	0.300	1.144	-0.356

Notes for Table 1:

(1) The ratio τ_{cyc}/σ'_{vc} from Vaid & Finn's results was selected at 3% strain reached in 10 cycles.

(2) The ratio τ_{cyc}/σ'_{vc} from Boulanger et al's results was selected at 3% strain reached in 15 cycles.

(3) A value of Q = 10 was used in Eq. (1) to calculate ξ_R for each test.

TABLE 2
CYCLIC TRIAXIAL TESTS
(After Boulanger, 2002)

Relative Density	σ'_{3c} (atm)	K_c	σ'_{fc} (atm)	τ'_{fc} (atm)	p' (atm)	τ_{cyc}/σ'_{fc}	α	K_α	ξ_R
Tests on Tailings Sand Conducted by Vaid & Chern (1985)									
0.7	2	1	2	0	2	0.194	0	1	-0.430
0.7	2	1.5	2.21	0.405	2.33	0.270	0.183	1.390	-0.418
0.7	2	2.0	2.41	0.809	2.67	0.347	0.335	1.786	-0.407
0.7	16	1	16	0	16	0.116	0	1	-0.084
0.7	16	1.25	16.8	1.62	17.3	0.133	0.096	1.151	-0.052
0.7	16	1.5	17.7	3.24	18.7	0.101	0.183	0.875	-0.019
0.7	16	2.0	19.3	6.47	21.3	0.050	0.335	0.431	0.049

Notes for Table 2:

(1) The ratio τ_{cyc}/σ'_{fc} from Vaid & Chem's results was selected at 2.5% strain reached in 10 cycles.

(2) A value of Q = 9 was used in Eq. (1) to calculate ξ_R for each test.

COMPARISONS OF K_α VALUES OBTAINED FROM CYCLIC TESTS WITH VALUES CALCULATED USING DERIVED EQUATIONS

The values of K_α calculated using Eq. (3) are first compared to those obtained from the cyclic simple shear tests of Vaid and Finn (1979) and Boulanger et al (1991). In as much as the cyclic simple shear test data were used to derive the equations, it would be expected that this comparison should provide excellent agreement, as suggested by the information provided in Fig. 3. The values of K_α obtained from the cyclic simple shear tests (Table 1) and the values calculated using Eq. (3) are listed in Table 3. These values are also plotted in Fig. 3 together with the derived curves for α = 0.1, 0.2, and 0.3. As can be noted, the calculated values are very close (in some cases almost identical) to those obtained from the cyclic simple shear tests. The derived curves represent an excellent fit to the test data for α = 0.1 and for α = 0.3, and a reasonable fit to the test data for α = 0.2.

Also shown in Fig. 3 are the derived curves for α = 0.05, 0.1, 0.15… 0.35 to illustrate the variations of K_α with the relative state parameter index ξ_R for a given value of α.

TABLE 3
COMPARING CALCULATED K_α VALUES AGAINST CYCLIC SIMPLE SHEAR DATA

Relative Density	From Cyclic Simple Shear Tests (Table 1)			Calculated K_α
	α	K_α	ξ_R	
Tests on Ottawa Sand Conducted by Vaid & Finn (1979)				
0.68	0	1	-0.486	1
0.68	0.093	1.294	-0.486	1.238
0.68	0.192	1.453	-0.486	1.542
0.50	0	1	-0.306	1
0.50	0.093	0.925	-0.306	0.959
0.50	0.192	0.879	-0.306	0.980
0.50	0.291	0.897	-0.306	0.960
Tests on Sacramento River Sand Conducted by Boulanger at al (1991)				
0.35	0	1	-0.156	1
0.35	0.100	0.952	-0.156	0.876
0.35	0.200	0.725	-0.156	0.788
0.35	0.300	0.635	-0.156	0.626
0.55	0	1	-0.356	1
0.55	0.100	0.981	-0.356	1.014
0.55	0.200	1.038	-0.356	1.095
0.55	0.300	1.144	-0.356	1.127

The values of K_α calculated using Eq. (3) were compared to those obtained from the cyclic triaxial tests of Vaid and Chern (1985). The values of K_α calculated using Eq. (3) and obtained from the tests are listed in Table 4. Using Q = 9, as suggested by Boulanger (2003a) for this tailings sand, the calculated K_α values are about 10 to 25 percent lower than those obtained from the triaxial test data, except for the test result at p' = 21.3 atm and α = 0.335 for which the calculated value is 43% lower. While slightly conservative for these tests, the values of K_α calculated using Eq. (3) capture the relative effect that increasing the confining stress from about

$2P_a$ to about $16P_a$ had on the same D_R = 0.7 tailings sand. The triaxial teats at $\alpha > 0$ were conducted at mean effective stresses greater than that used for the tests at $\alpha = 0$. Thus, some of these differences, between the calculated K_α and K_α based on the results of the triaxial tests, may be attributable to this difference in mean effective stresses (i.e., K_σ effect).

Values of K_α calculated with Q = 9.4 are also presented in Table 4. These calculated K_α values range from 15% lower to 11% higher than the triaxial data, except for the test at p' = 17.3 atm and α = 0.096 where the calculated value is 23% lower. The effects of Q on K_α are illustrated and discussed further in a later section of the paper (see Table 6).

TABLE 4
COMPARING CALCULATED K_α VALUES AGAINST HIGH CONFINING STRESS TESTS

Relative Density	From Cyclic triaxial Tests (Table 2)				Calculated K_α for Q=9	Calculated K_α for Q=9.4
	p' (atm)	α	K_α	ξ_R		
Tests on Ottawa Sand Conducted by Vaid & Chern (1985)						
0.7	2	0	1	-0.430	1	1
0.7	2.33	0.183	1.390	-0.418	1.249	1.350
0.7	2.67	0.335	1.786	-0.407	1.363	1.525
0.7	16	0	1	-0.084	1	1
0.7	17.3	0.096	1.151	-0.052	0.855	0.889
0.7	18.7	0.183	0.875	-0.019	0.733	0.810
0.7	21.3	0.335	0.431	0.049	0.247	0.478

INFLUENCE OF MEAN EFFECTIVE NORMAL STRESS AND RELATIVE DENSITY ON K_α

The variations of K_α with mean effective normal stress, p', are shown in Fig. 4 considering a soil having a relative density of D_R = 0.4 and Q = 10. Similar results are presented in Fig. 5 for a soil having a relative density of D_R = 0.7 and Q = 10. The results shown in Figs. 4 and 5 are for p'/P_a = ½, 1, 2, 4, 8 and 16. These two figures illustrate the strong influence on K_α of both the mean effective normal stress and the relative density of the soil. This influence is also illustrated by the values listed in Table 5, which indicate a decrease in K_α as p'/P_a increases for all values of α and for both D_R = 0.4 and D_R = 0.7. The value of K_α increases with an increase in relative density, all other conditions being the same.

TABLE 5
EFFECT OF MEAN EFFECTIVE NORMAL STRESS AND RELATIVE DENSITY ON K_α VALUES

p'/P_a	K_α at α = 0.1		K_α at α = 0.2		K_α at α = 0.3	
	D_R = 0.4	D_R = 0.7	D_R = 0.4	D_R = 0.7	D_R = 0.4	D_R = 0.7
1	0.899	1.351	0.844	1.717	0.729	1.933
2	0.887	1.265	0.815	1.567	0.677	1.752
8	0.859	1.071	0.745	1.208	0.541	1.288
16	0.845	0.971	0.705	1.005	0.455	0.994

INFLUENCE OF THE PARAMETER Q ON K_α

The values of K_α at p'/P_a = 1 and 8 with Q = 8, 9, and 10 calculated using Eqs. (2) and (3) are listed in Table 6. These results indicate that soils with a higher Q would have a higher value of K_α, especially at high values of α. The results also indicate that the relative effect increases somewhat at higher relative densities and at higher values of the mean effective normal stress.

As noted by Bolton (1986), the parameter Q depends on the grain type and is typically equal to 10 for quartz and feldspar, 8 for limestone, 7 for anthracite, and 5.5 for chalk. Accordingly, for most soils within an embankments or the foundation of an embankment, a value of Q = 9 or 10 would seem reasonable.

TABLE 6
EFFECT OF PARAMETER Q ON K_α VALUES

	K_α at p'/P_a = 1					
	D_R = 0.4			D_R = 0.7		
α	Q = 8	Q = 9	Q = 10	Q = 8	Q = 9	Q = 10
0.1	0.861	0.881	0.899	1.083	1.224	1.351
0.2	0.749	0.801	0.844	1.231	1.494	1.717
0.3	0.550	0.650	0.729	1.319	1.663	1.933
	K_α at p'/P_a = 8					
	D_R = 0.4			D_R = 0.7		
α	Q = 8	Q = 9	Q = 10	Q = 8	Q = 9	Q = 10
0.1	0.826	0.839	0.859	0.837	0.930	1.071
0.2	0.642	0.688	0.745	0.680	0.915	1.208
0.3	0.279	0.413	0.541	0.393	0.851	1.288

RELATING K_α TO SPT BLOW COUNT

The relative state parameter index, ξ_R, is expressed in Eq. (2) as a function of mean effective normal stress, relative density and the parameter Q. Equation (2) can be rewritten by substituting an expression that relates relative density to SPT blow count. Over the past several decades, many researchers have proposed expressions relating SPT blow count to relative density of a cohesionless soil. The relationship developed by Meyerhof (1957) is typical of most available expressions and is given by:

$$N = \left(17 + 24\frac{\sigma'_{vo}}{P_a}\right)D_R^2 \qquad (4)$$

in which N is the SPT blow count (in blows per ft) taken at a depth having an effective vertical stress σ'_{vo}, and D_R (in decimal) is the relative density. The SPT blowcount adjusted for an effective vertical stress of one atmosphere is designated N_1. Thus, Eq. (4) can be rewritten as follows:

$$N_1 = (a + b) D_R^2 \tag{5}$$

The sum $(a + b)$ = 41 in the original Meyerhof relationship [Eq. (4)]. As noted by Cubrinovski and Ishihara (1999), the sum $(a + b)$ is affected by the grain size characteristics and the type of soil under consideration. Cubrinovski and Ishihara included data for high quality undisturbed samples (obtained by in situ freezing) for clean sand and for silty sand. The relative density, fines content, N_1, and median grain size, D_{50}, of each undisturbed sample are tabulated by Cubrinovski and Ishihara, and can be used to calculate the sum $(a + b)$. The calculated values of $(a + b)$ are plotted in Fig. 6, which indicate that the sum $(a + b)$ is higher for the clean sands than it is for the silty sands, but is very weakly dependent on D_{50}. The average values of the sum $(a + b)$ for the soils included in Fig. 6 are summarized in Table 7.

TABLE 7

AVERAGE $(a + b)$ VALUES USING DATA BY CUBRINOVSKI AND ISHIHARA (1999)

Samples	average $(a + b)$**
Silty Sand Samples	19.7
Clean Sand Samples	38.9
All Samples	29.9

** Using N_1 values reported by Cubrinovski and Ishihara (1999)

It may be noted that the SPT blow counts used for calculating the sum $(a + b)$ for the samples presented in Fig. 6 were most likely obtained with a delivered energy of about 80%. If the usual adjustment is made (i.e., multiplying each blow count by the ratio 80/60) to convert the SPT blow counts tabulated by Cubrinovski and Ishihara (1999) to equivalent values of $(N_1)_{60}$, the above averages of $(a + b)$ would be those shown in Table 8.

TABLE 8

AVERAGE $(a + b)$ VALUES AFTER CONVERSION TO 60% ENERGY RATIO

Samples	average $(a + b)$***
Silty Sand Samples	26.2
Clean Sand Samples	51.9
All Samples	39.9

***Values of N_1 reported by Cubrinovski and Ishihara (1999) were multiplied by the ratio 80/60 to convert to $(N_1)_{60}$

It is interesting to note that the value of $(a + b)$ = 39.9 for all samples is very close to that obtained from the original Meyerhof relationship [Eq. (4)].

If the value of $(a + b)$ = 51.9 were used for clean sands, a relative density of only about 76% is obtained for an SPT blow count $(N_1)_{60}$ = 30, which is the limiting value for triggering liquefaction in a clean sand (fines content ≤ 5%) in the currently used SPT-based liquefaction evaluation procedure. A more realistic value for $(a+b)$ might be 46, which would result in a relative density of about 81% for an SPT blow count $(N_1)_{60}$ = 30. Therefore, a value of $(a+b)$ = 46 seems reasonable to use for clean sands and will be adopted for estimating the variations of K_α with $(N_1)_{60}$.

Equation (2) can be rewritten for clean sands in terms of $(N_1)_{60}$ as follows:

$$\xi_R = \frac{1}{Q - Ln\left(\frac{100p'}{P_a}\right)} - \sqrt{\frac{(N_1)_{60}}{46}} \tag{6}$$

The effective vertical stress, σ'_{vo}, is more widely used in practice and Eq. (6) can be rewritten in terms of σ'_{vo} and the lateral earth pressure coefficient at rest, K_o, as follows:

$$\xi_R = \frac{1}{Q - Ln\left(\frac{100(1+2K_o)\sigma'_{vo}}{3P_a}\right)} - \sqrt{\frac{(N_1)_{60}}{46}} \tag{7}$$

The values of K_α can then be calculated for any given set of $(N_1)_{60}$ and σ'_{vo}/P_a using Eqs. (7) and (3). Values of K_α at σ'_{vo}/P_a = 1 for $(N_1)_{60}$ = 4, 8, 12, 16, and 20 are presented in Fig. 7 and those at σ'_{vo}/P_a = 4 and the same SPT blow counts are presented in Fig. 8. A value of K_o = 0.45 was used for generating these curves. The effects of K_o are covered later in this paper.

Curves such as those presented in Figs. 7 and 8 can be used to estimate K_α, and hence, the cyclic resistance of a soil layer beneath a sloping ground. It is suggested that an equivalent clean sand SPT blow count (i.e., $(N_1)_{60cs}$) be used in Eq. (7) for cohesionless soils with fines contents greater than 5%.

RELATING K_α TO CPT TIP RESISTANCE

Boulanger (2002, 2003b) summarized the solutions completed by Salgado et al (1997a, 1997b) for CPT tip resistance and suggested that these solutions are closely approximated by:

$$\frac{q_c}{P_a} = C_o C_1 \left(\frac{\sigma'_{vo}}{P_a}\right)^m \left(\frac{K_o}{0.45}\right)^{m-0.077} \tag{8a}$$

$$m = 0.7836 - 0.5208D_R \tag{8b}$$

$$C_o = 25.7 + 39.7D_R + 212.3D_R^2 \tag{8c}$$

C_1 = 1 typical soil property set

C_1 = 0.64 lower bound soil property set (8d)

C_1 = 1.55 upper bound soil property set in which q_c is the CPT tip resistance, and K_o, P_a and $\sigma_{vo}^{'}$ are as defined earlier in this paper.

The corrected CPT tip resistance (i.e., tip resistance corresponding to an effective vertical stress, $\sigma_{vo}^{'}$ = atmosphere) is given by:

$$q_{c1} = C_q q_c \tag{9}$$

The coefficient C_q can be expressed as follows:

$$C_q = \left(\frac{P_a}{\sigma_{vo}^{'}} \right)^m \tag{10}$$

in which m varies with D_R as provided in Eq. (8b). The use of a normalized (i.e., dimensionless) corrected tip resistance, q_{c1N}, was suggested by Robertson and Wride (1997) and is equal to:

$$q_{c1N} = \frac{q_{c1}}{P_a} = \left(\frac{P_a}{\sigma_{vo}^{'}} \right)^m \left(\frac{q_c}{P_a} \right) \tag{11}$$

Combining Eqs. (8a), (8c) and (11), provides the following relationship for q_{c1N} as a function of relative density and the coefficient of lateral pressure at rest, K_o:

$$q_{c1N} = C_1 \left(25.7 + 39.7 D_R + 212.3 D_R^2 \right) \left(\frac{K_o}{0.45} \right)^{m-0.077} \tag{12}$$

The value of the coefficient $\left(K_o/0.45\right)^{m-0.077}$ depends on relative density and on K_o. The variation of this coefficient can have for a range of D_R and K_o values are illustrated in Table 9.

TABLE 9

VARIATION OF COEFFICIENT WITH D_R AND K_o

D_R	m	K_o	$\left(K_o/0.45\right)^{m-0.077}$
All relative densities	---	0.45	1.00
0.4	0.575	0.50	1.05
0.5	0.523	0.50	1.05
0.6	0.471	0.50	1.04
0.8	0.367	0.50	1.03
0.5	0.523	0.55	1.09
0.6	0.471	0.55	1.08
0.8	0.367	0.55	1.06
0.6	0.471	0.60	1.12
0.8	0.367	0.60	1.09
0.6	0.471	0.70	1.19
0.8	0.367	0.70	1.14

Thus, the product $C_1\left(K_o/0.45\right)^{m-0.077}$ can vary from 0.64 for the lower bound property set with K_o = 0.45 to possibly over the 1.7 for the upper bound property set, K_o = 0.6 and D_R = 0.6. The wide range of possible values for this product produces a wide variation in the relation between q_{c1N} and D_R described by Eq. (12). This wide variation is, however, similar to the variation observed between $\left(N_1\right)_{60}$ and D_R, as represented by the data in Fig. 6.

The product $C_1\left(K_o/0.45\right)^{m-0.077}$ can, however, be reasonably selected to result in a relative density of about 80% at the limiting value of q_{c1N} to trigger liquefaction. The latter value for most of the currently available relationships is $\left(q_{c1N}\right)_{Lim}$ = 175±. Use of a value for $C_1\left(K_o/0.45\right)^{m-0.077} \approx 0.9$ in Eq. (12) results in a value of q_{c1N} = 175 at D_R = 0.8.

Equation (12) can then be rewritten as follows:

$$q_{c1N} = 23.1 + 35.7D_R + 191.1D_R^2 \qquad (13)$$

Equation (13) can be inverted to provide an equation relating relative density to the corrected normalized CPT tip resistance. The following approximation is derived:

$$D_R = 0.086\sqrt{q_{c1N}} - 0.334 \qquad (14)$$

This approach provides the means to calculate the relative state parameter index in terms of q_{c1N} by substituting Eq. (14) into Eq. (2). Thus:

$$\xi_R = \frac{1}{Q - Ln\left(\dfrac{100p'}{P_a}\right)} - \left(0.086\sqrt{q_{c1N}} - 0.334\right) \qquad (15)$$

The latter equation can also be rewritten in terms of the effective vertical stress, i.e.:

$$\xi_R = \frac{1}{Q - Ln\left(\dfrac{100\left(1+2K_o\right)\sigma'_{vo}}{3P_a}\right)} - \left(0.086\sqrt{q_{c1N}} - 0.334\right) \qquad (16)$$

The values of K_α can then be calculated for any given set of q_{c1N} and σ'_{vo}/P_a using Eqs. (16) and (3). Values of K_α at σ'_{vo}/P_a = 1 for q_{c1N} = 60, 80, 100, 120, and 160 are presented in Fig. 9 and those at σ'_{vo}/P_a = 4 and the same CPT tip resistances are presented in Fig. 10. A value of K_o = 0.45 was used for generating these curves. The influence of K_o on these calculations is covered in the next section.

INFLUENCE OF LATERAL EARTH PRESSURE COEFFICIENT AT REST, K_o

A value of the lateral earth pressure coefficient at rest, K_o, is required to calculate the relative state parameter index using Eq. (7) or Eq. (16). The variations of K_α with effective vertical stress and $(N_1)_{60}$ as illustrated in Figs. 7 and 8, or q_{c1N} as illustrated in Figs. 9 and 10, were calculated using a value of K_o = 0.45.

To evaluate the influence of K_o on K_α, values of K_α were calculated for σ'_{vo}/P_a = 1 and 4 and K_o = 0.45 and 0.60 using Eqs. (7) and (3) with $(N_1)_{60}$ = 4, 12 and 20. The results are presented in Figs. 11a and 11b. The values of K_α for σ'_{vo}/P_a = 1 and 4 and K_o = 0.45 and 0.60 using Eqs. (16) and (3) with q_{c1N} = 60, 100 and 120 are presented in Figs. 11c and 11d.

The information presented in Fig. 11 indicates that the lateral earth pressure coefficient at rest, K_o, has little or negligible influence on the calculated values of K_α.

CONCLUDING REMARKS

The relations derived in this paper provide a practical means for estimating the static shear stress ratio correction factor, K_α, for use in evaluating the cyclic resistance of soils beneath sloping ground. The relative state parameter index, ξ_R, provided the framework for describing the experimentally observed dependence of K_α on both relative density, D_R and effective confining pressure, σ'_{vo}. Relations were derived that express K_α as a function of the static shear stress ratio, the mean effective confining pressure, the effective vertical pressure, and the soil's denseness (expressed in terms of D_R, normalized SPT blow count, or normalized CPT tip resistance).

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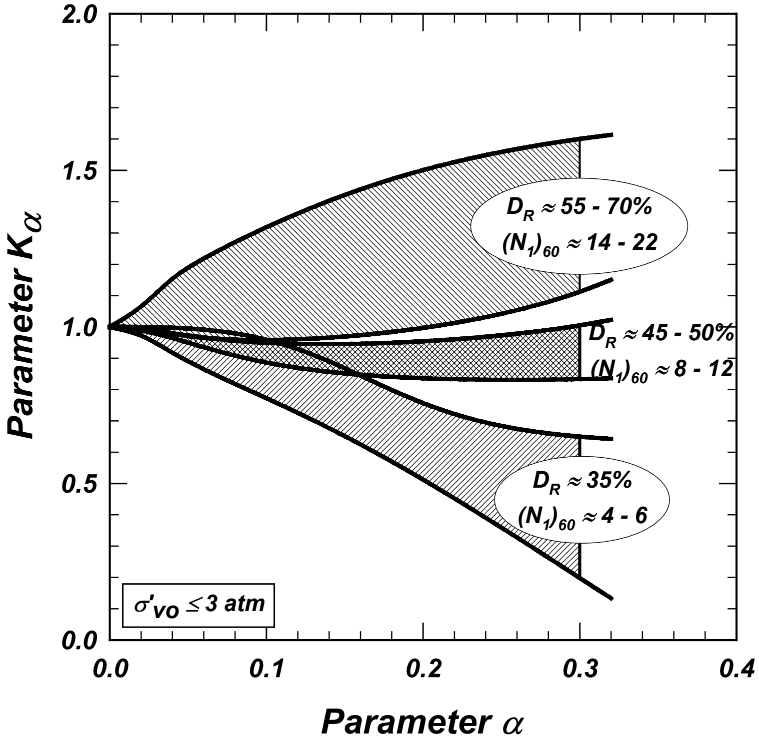

Fig. 1 Values of K_α Recommended by Harder and Boulanger (1997) for Vertical Effective Confining Pressures Less than 3 Atmospheres

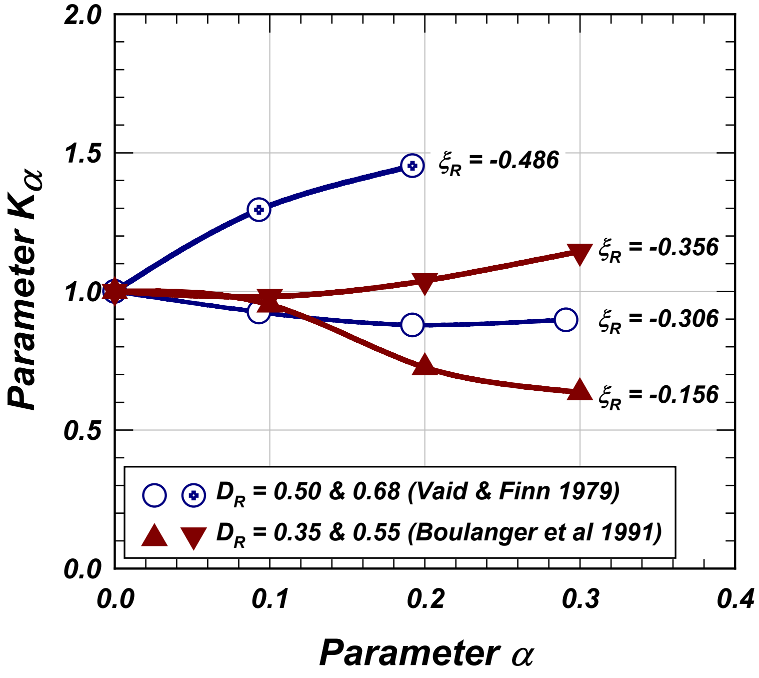

Fig. 2 Cyclic Simple Shear Test Data for a Range of D_R Values at an Effective Confining Pressure of 2 Atmospheres

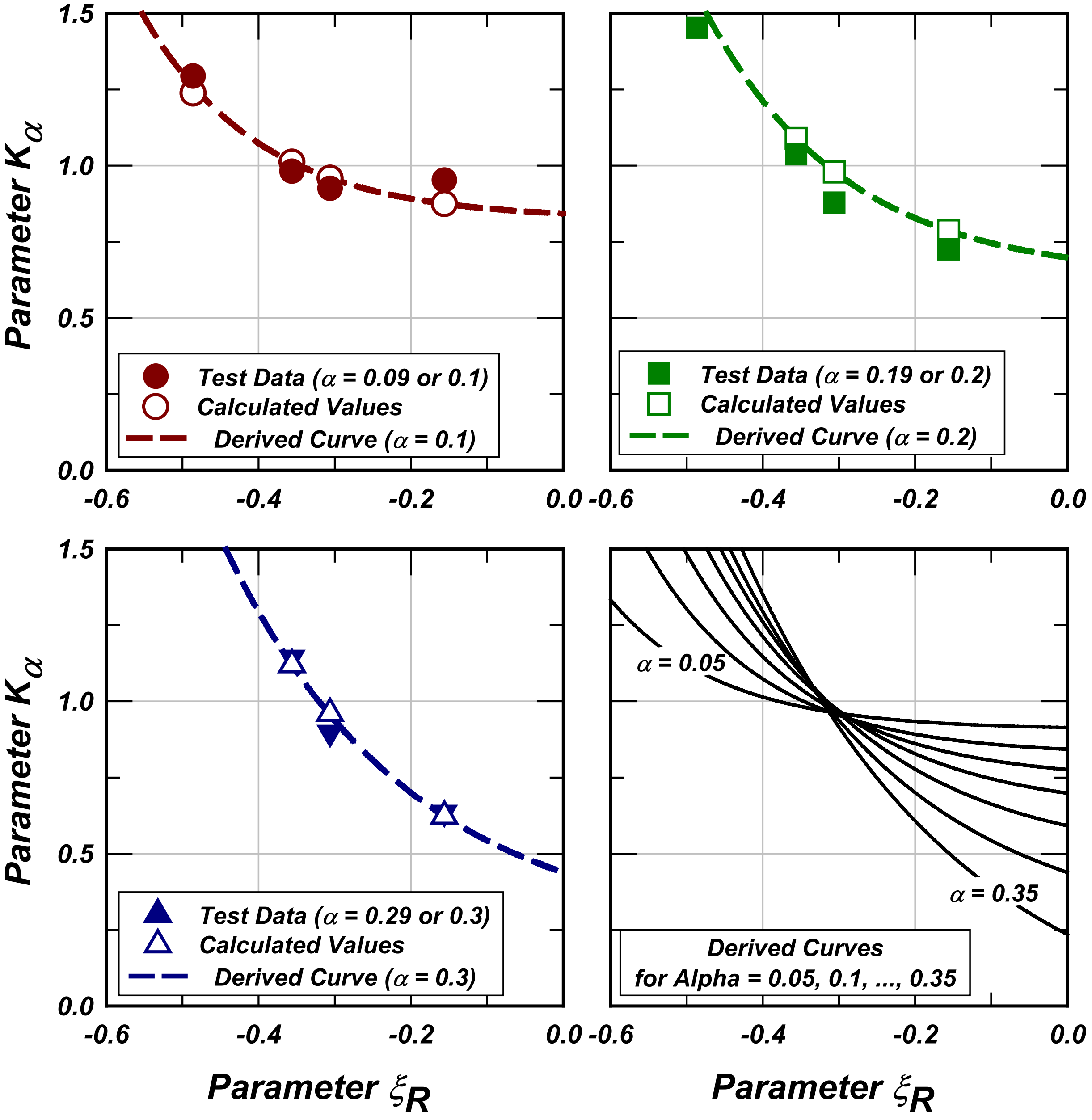

Fig. 3 Comparison of Derived Expressions of (K_α) versus (ξ_R) with Values Obtained from Simple Shear Tests (Table 1) and Corresponding Values Calculated Using Eq. 3 (Table 3)

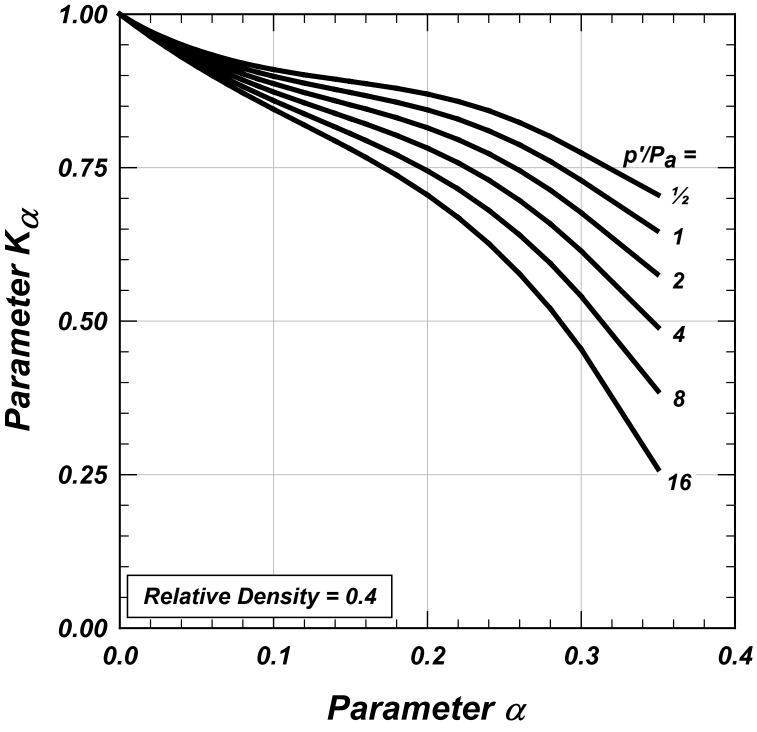

Fig. 4 Influence of $p^{'}/P_a$ on K_α for a Soil with a Relative Density of 40% and Q=10

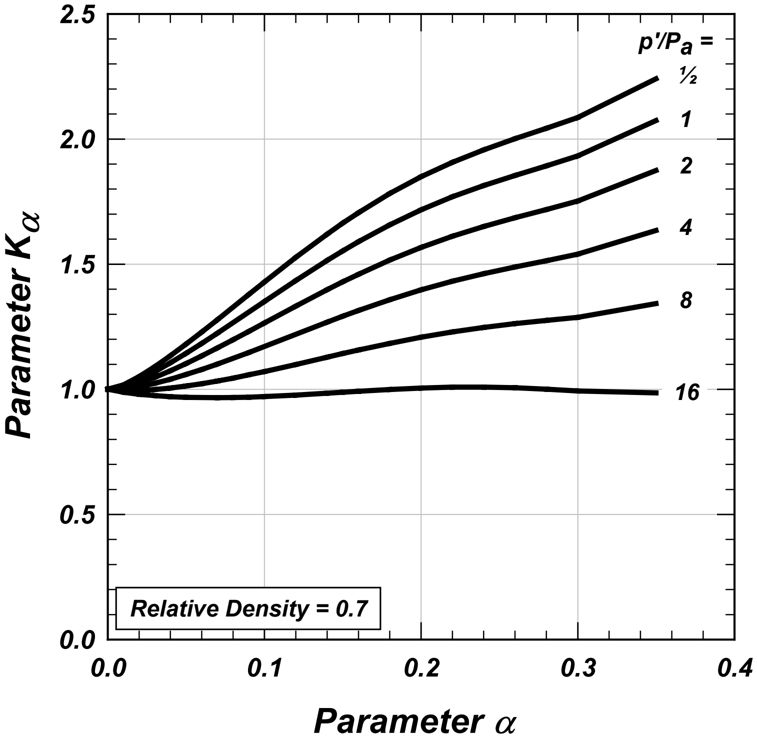

Fig. 5 Influence of $p^{'}/P_a$ on K_α for a Soil with a Relative Density of 70% and Q=10

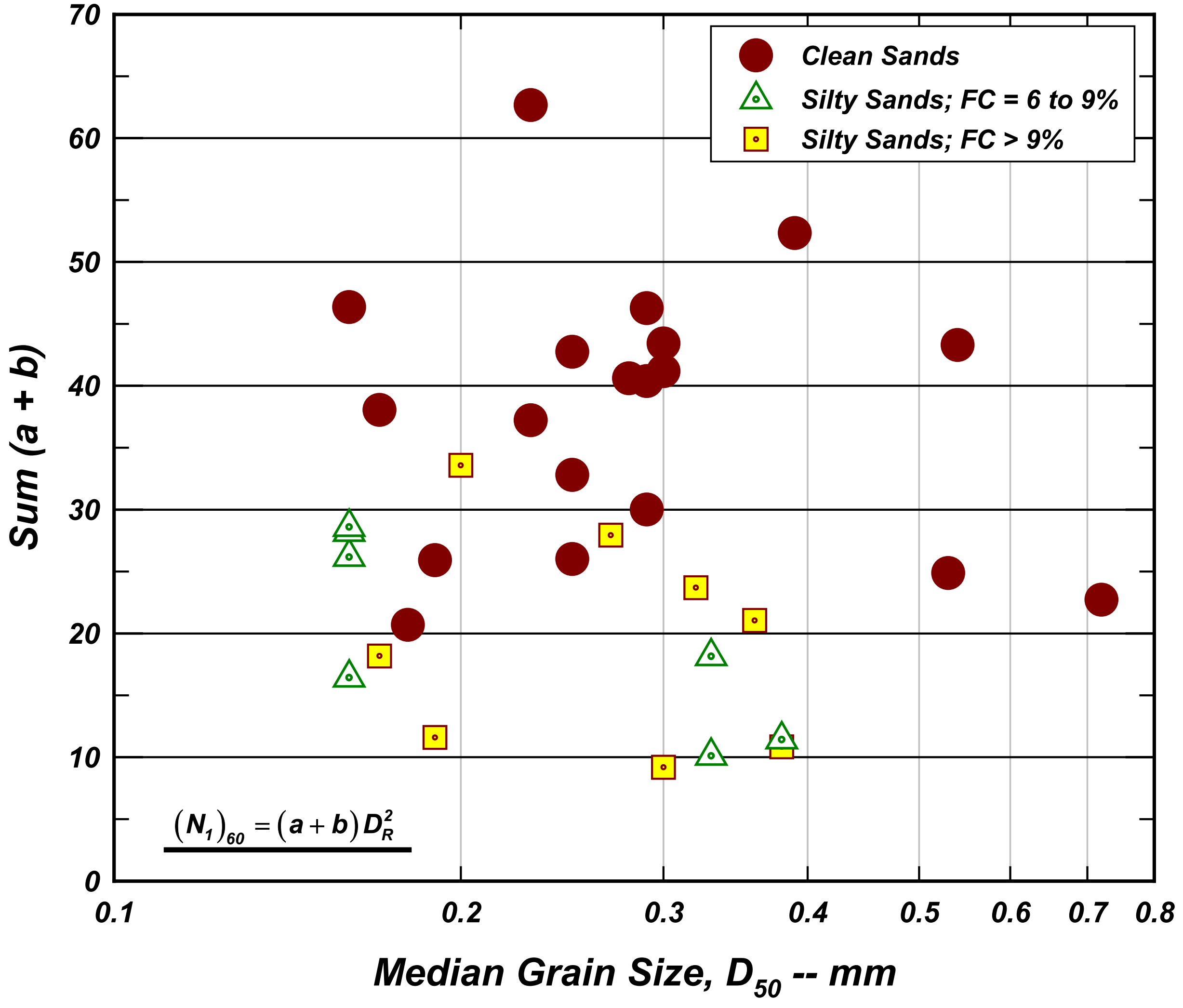

Fig. 6 Variations of the Sum $(a+b)$ with D_{50}

[Values of relative density, N_1, FC, and D_{50} are from Cubrinovski and Ishihara (1999)]

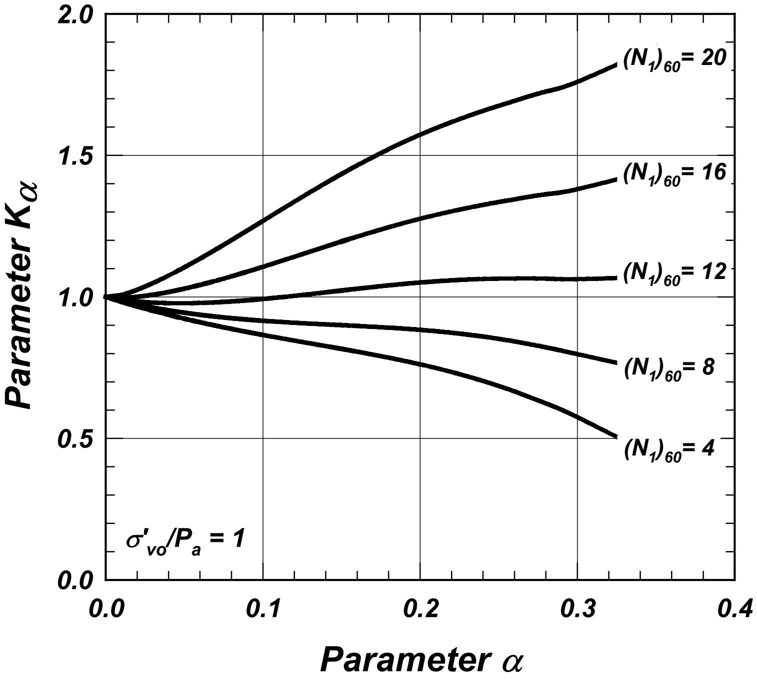

Fig. 7 Variations of K_α with SPT Blow Count $(N_1)_{60}$ at an Effective Vertical Stress, σ'_{vo} = 1 atmosphere

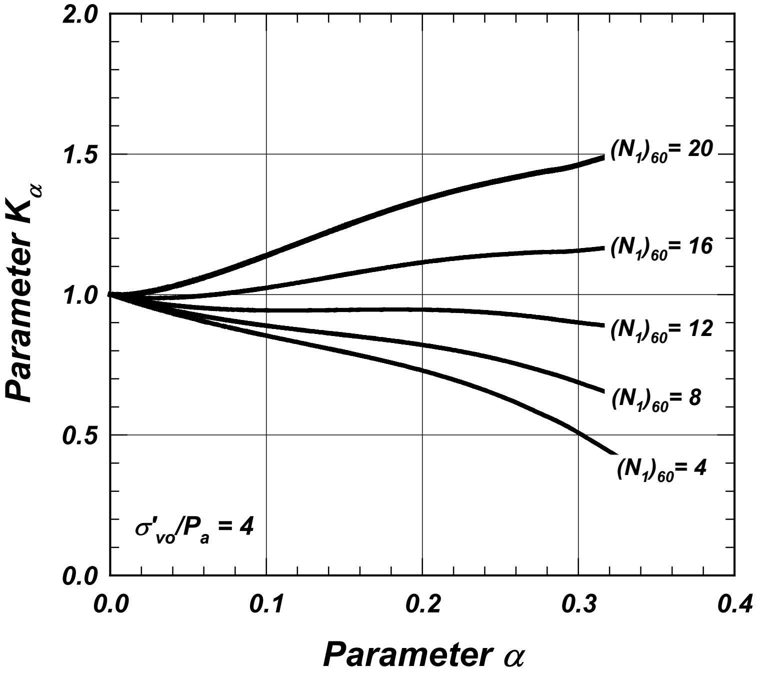

Fig. 8 Variations of K_α with SPT Blow Count $(N_1)_{60}$ at an Effective Vertical Stress, σ'_{vo} = 4 atmosphere

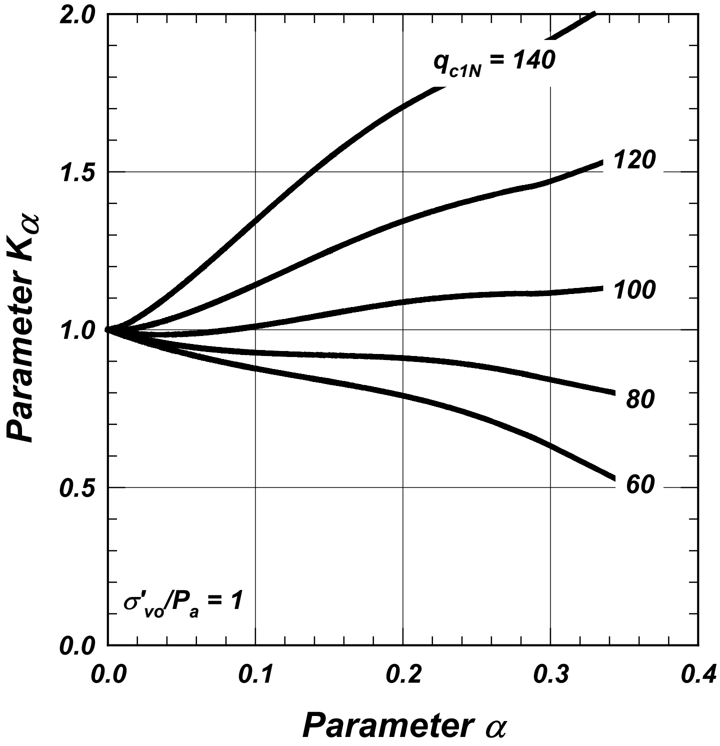

Fig. 9 Variations of K_α with CPT Normalized Tip Resistance q_{c1N} at an Effective Vertical Stress, σ'_{vo} = 1 atmosphere

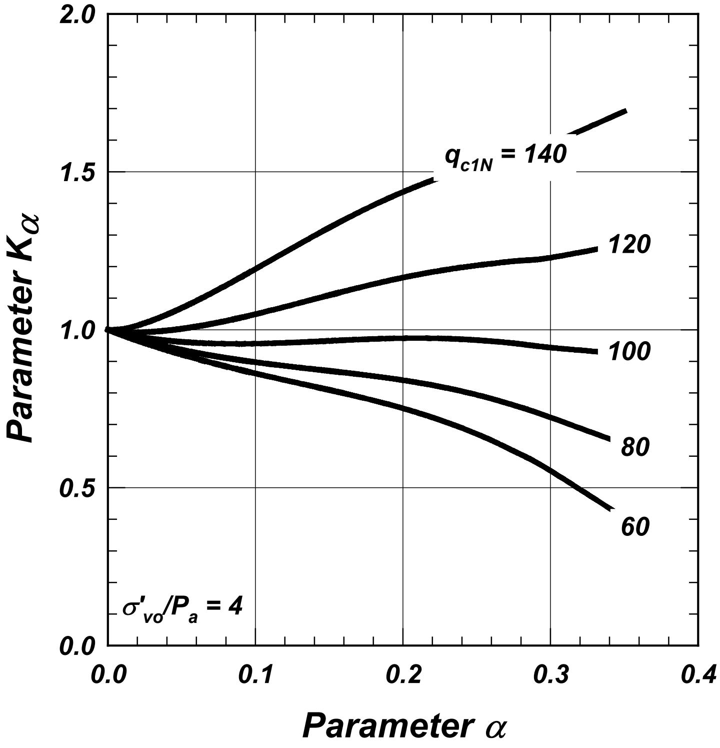

Fig. 10 Variations of K_α with CPT Normalized Tip Resistance q_{c1N} at an Effective Vertical Stress, σ'_{vo} = 4 atmosphere

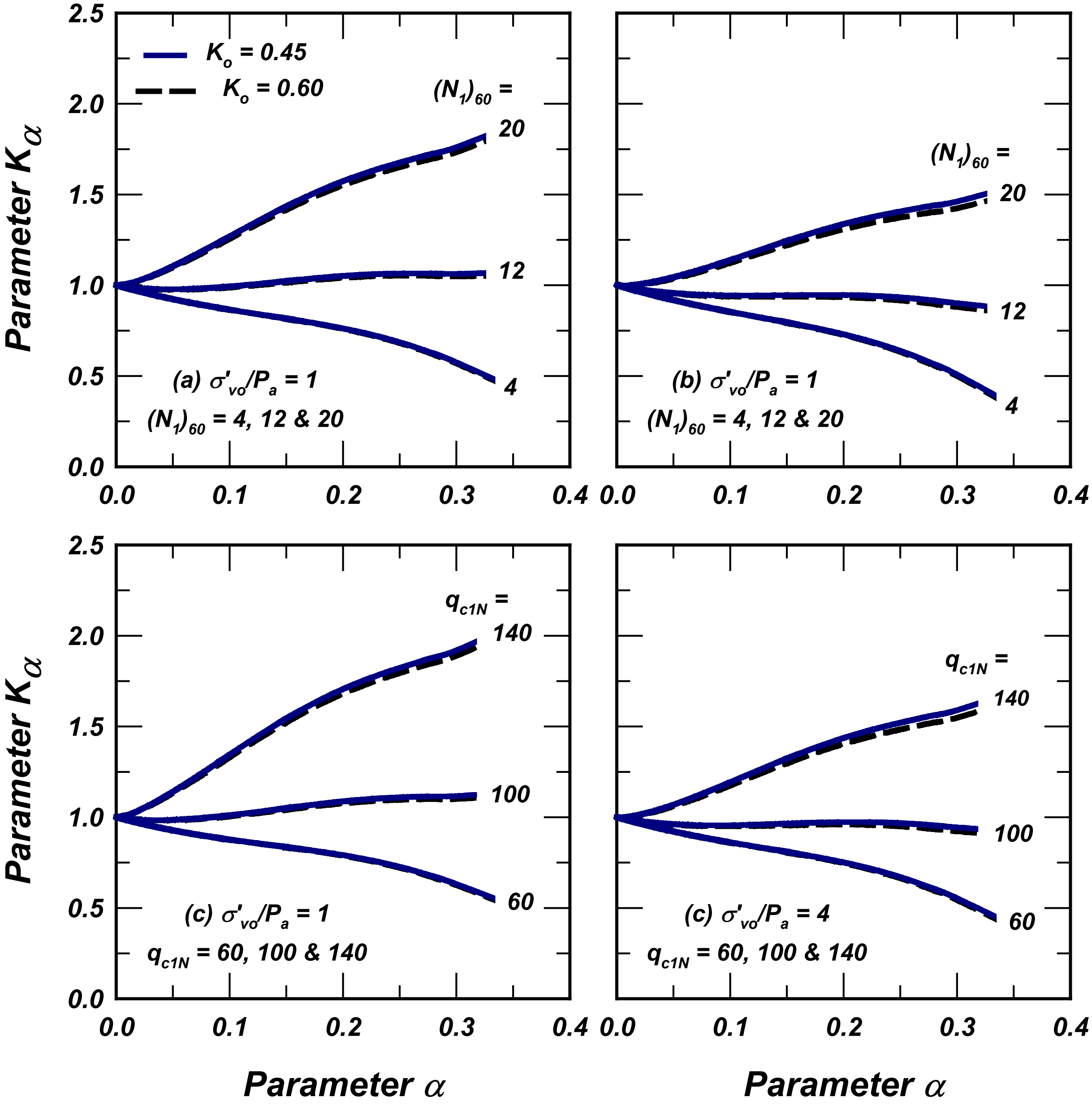

Fig. 11 Influence of K_o on K_α